Comparisons of Free-Flight and Wind-Tunnel Data on Slender Cones

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Nomenclature

C_H = heat-transfer coefficient
C_p = pressure coefficient
C_∞ = form of Chapman-Rubesin viscosity coefficient, $\mu_w/\mu_\infty = C_\infty T_w/T_\infty$
C^* = form of Chapman-Rubesin viscosity coefficient (see Ref. 6)
d = nose diameter
K = nose drag coefficient
T = temperature
α = angle of attack, rad
γ = ratio of specific heats
ϵ = $(\gamma - 1)/(\gamma + 1)$
μ = gas viscosity
ψ = nose radius/base radius

Subscripts

d = base on nose diameter
0 = stagnation conditions
w = wall conditions
∞ = freestream conditions

THE role of the near-perfect gas hypersonic wind tunnel in the development of missiles and aircraft often has been hampered by questions of support interference and real-gas effects. The purpose of this note is to demonstrate by a comparison of data the apparent lack of support interference effects on forebody flow fields of slender cones and that so-called real-gas effects on the stability, drag, and heat-transfer rates for slender cones are insignificant at velocities up to 18,000 fps.

Figures 1 and 2 are correlations (following Whitfield and Wolny[1]) of the experimental normal force and pitching moment data on 8° and 9° half-angle (θ_c) slender cones from the Naval Ordnance Laboratory (NOL) aeroballistic range[2] and the AEDC-VKF 100-in. hypersonic Mach 20 tunnel (F), respectively. These data are relatively insensitive to Mach number M_∞ and Reynolds number Re, per se, and therefore they offer some assessment of real-gas and/or support interference effects. A comparison of the NOL and AEDC-VKF data indicates that the free-flight data at velocities (U_∞) up to 18,000 fps are in excellent agreement with the essentially perfect gas sting mounted wind-tunnel data. Also worthy of note is the fact that both sets of data agree with previously published data.[1, 3]

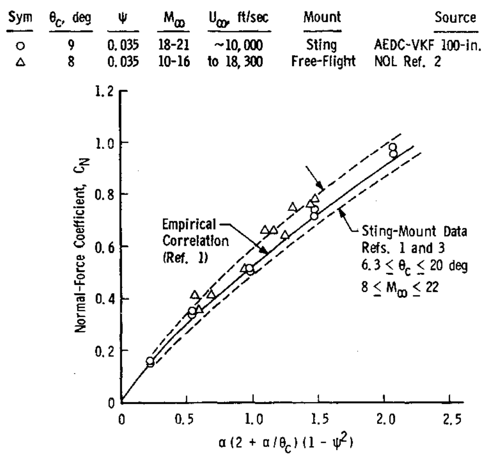

Fig. 1 Comparison of sting-mount and free-flight normal force data.

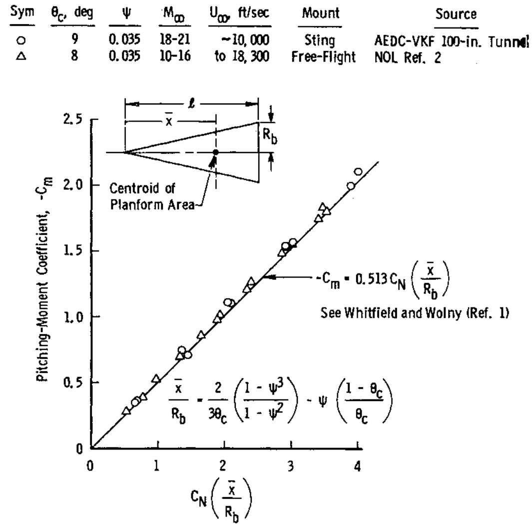

Fig. 2 Comparison of sting-mount and free-flight pitching-moment data.

A summary of drag data from slender cones obtained with free-flight and balance techniques is shown in Fig. 3. The viscous drag coefficient ΔC_{D_v} (total forebody drag less inviscid pressure drag) is plotted vs the hypersonic viscous parameter $\bar{v}_\infty$. Note that the cold-wall NOL-range data agree with previously published[4] high Mach number sting-mounted data. The AEDC-VKF free-flight data[4] ($T_w/T_0 \approx 0.33$) were taken in the 50-in. Mach 10 tunnel by launching models from a pneumatic launch tube obtained through the courtesy of Bain Dayman (Jet Propulsion Laboratories). The agreement of the range free-flight data and the previous balance data and the consistent trend of the Mach 10 free-flight and balance data (with T_w/T_0) certainly indicate no appreciable support interference or real-gas effects. Recent slender cone free-flight data published by Dayman[5] indicate greater viscous effects than the present data. No reason can be noted at present for the apparent discrepancy between the data reported by Dayman and the present data.

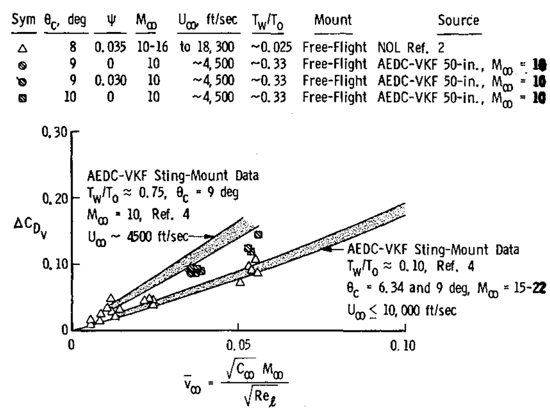

Fig. 3 Comparison of sting-mount and free-flight viscous drag data, $\alpha = 0°$.

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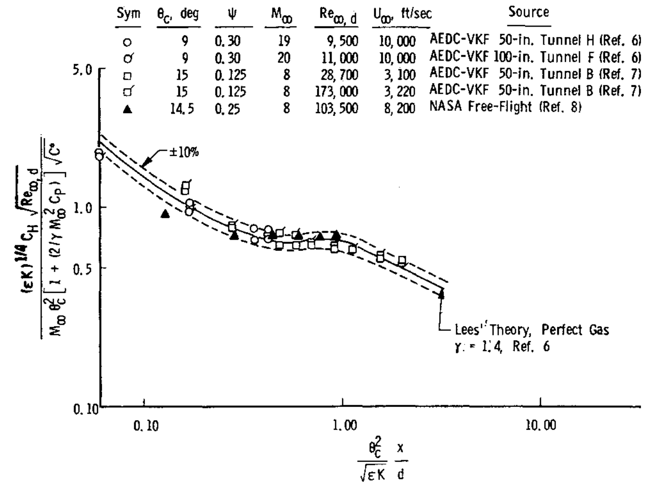

Fig. 4 Correlation of heat-transfer distribution to spherically blunted cones.

Figure 4 presents a correlation (following Griffith and Lewis[6]) of experimental and theoretical heat-transfer data over slender cones. The data presented represent data[7] from the AEDC-VKF 50-in. Mach 8 tunnel (B), data[6] from the AEDC-VKF 50- and 100-in. Mach 20 tunnels (H and F) and free-flight data[8] taken by NASA (NACA) personnel on the conical nose region of a spacecraft configuration. This correlation again indicates that support interference effects on the forebody are insignificant in the flight regime represented by these data.

In conclusion, the comparisons with free-flight data indicate that, for some situations of interest, real-gas effects are insignificant at velocities up to 18,000 fps and that support interference effects are negligible. Therefore, in this speed range it is possible to provide significant flight simulation in terms of Mach and Reynolds numbers alone, i.e., in existing test facilities that operate in the velocity regime of 10,000 fps.

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Reduction of Stiffness and Mass Matrices

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Just as it is often necessary to reduce the size of the stiffness matrix in statical structural analysis, the simultaneous reduction of the nondiagonal mass matrix for natural mode analysis may also be required. The basis for one such reduction technique may follow the procedure used in Ref. 1 for the stiffness matrix, namely, the elimination of coordinates at which no forces are applied.

Arrange the structural equations $\{F\} = [K]\{x\}$ so that after partitioning in the form

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} A & B \\ B' & C \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

the forces F_2 are to be zero. The two resulting equations yield

$$F_1 = (A - BC^{-1}B')x_1$$

from which the reduced stiffness matrix is seen to be

$$K_1 = A - BC^{-1}B'$$

The foregoing amounts to a coordinate transformation $x = Tx_1$ or

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} I \\ -C^{-1}B' \end{bmatrix} \{x_1\}$$

If the structure energies are written $T = \frac{1}{2}\dot{x}'M\dot{x}$ and $V = \frac{1}{2}x'Kx$ and the foregoing transformation is employed, the result is

$$T = \tfrac{1}{2}\dot{x}_1{}'T'MT\dot{x}_1$$

$$V = \tfrac{1}{2}x_1{}'T'KTx_1$$

The reduced stiffness matrix is seen to be $K_1 = T'KT$ and the reduced mass matrix $M_1 = T'MT$. Then with

$$[M] = \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{B}' & \bar{C} \end{bmatrix}$$

the reduced mass matrix becomes

$$M_1 = \bar{A} - \bar{B}C^{-1}B' - (C^{-1}B')'\,(\bar{B}' - \bar{C}C^{-1}B')$$

In the case of the reduced stiffness matrix, none of the structural complexity is lost since all elements of the original stiffness matrix contribute. However, in the reduced mass matrix, combinations of stiffness and mass elements appear. The result is that the eigenvalue-eigenvector problem is closely but not exactly preserved. Some comparative results are reported in Ref. 2 for beam vibrations.

References

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